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Diamagnetic interactions of superheated-superconducting spheres in a bidimensional lattice

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Abstract. In order to simulate a Superheated-Superconducting detector which is under construction for Particle Physics purposes, we study a bidimensional lattice of small superconducting spheres placed in an external magnetic field. We propose a model to study the diamagnetic interactions among the spheres and solve it using numerical Monte-Carlo techniques. New phenomena are found and the ensuing results are analyzed. Finally it is proposed a qualitative explanation.

1 Introduction

The particle physics tendencies in the last years need of new non conventional detection methods which are presently under development. Among them, the use of superheated- superconducting granules (SSG) has been proposed for photon, dark matter and neutrinos detection [1] The SSG detectors are made of small grains (of a few microns of diameter) of type I superconducting metals which undergo a first order phase transition between normal and superconducting states. As in any other first order phase transition, metastable states can be reached if for a fixed temperature (below the critical one) the strength of the applied magnetic field $\boldsymbol{B} \equiv \boldsymbol{B}$ varies between $B_{sc} \equiv |\boldsymbol{B}_{sc}|$ and $B_{sh} \equiv |\boldsymbol{B}_{sh}|$. Starting with a normal conducting sample in an external magnetic field $B > B_c \equiv |\boldsymbol{B}_c|$ and decreasing the strength of $\boldsymbol{B}$, the sample remains in a metastable normal state (supercooled) until certain value $B = B_{sc} < B_c$ is reached; at this point the transition to the superconducting state occurs. In the same way, starting from a superconducting sample, the field strength can be increased above B_c without phase transition and the sample remains in a metastable superconducting state (superheated) until certain value $B = B_{sh} > B_c$ is reached; at this point the sample becomes normal conducting.

As any other metastable state, the superheated state is very sensitive to the existing defects at the surface of the superconductor, however for very small grains with an extremely carefully prepared surface it is possible to obtain the superheated state in externally applied magnetic fields very near of B_{sh}. For such grains, the deposition of a few KeV of energy into a given grain will flip it into the normal state, thereby producing a change of magnetic flux which can be detected in a pickup coil connected to a SQUID or other sensitive circuit. In this way, SSG detectors act as threshold devices, and the energy information must be obtained by making different measurements at different applied external fields and differentiating the results. Furthermore, by using an appropriated lattice of pickup coils, the spatial information about the particles trajectories can be stored, improving the efficiency of the detector.

The fact that a SSG detector can be sensible to the deposition of energies so low as a few KeV. is important specially for neutrino detection, where a SSG based detector could be sensible to coherent neutral current neutrinonucleus scattering [2], where the cross section is about 1000 times larger than that of other processes like inverse beta decay; thus a SSG detector with a few kilograms would measure the same event rate as a multiton detector based on other processes. For real detectors, in order to discriminate between interesting and non interesting events taking place in the detector, it is very important that one can simulate the physics of the detector which in the present case is limited by the existence of diamagnetic interactions between the superconducting spheres. There is little theoretical and experimental information about this kind of interactions so that we propose a simplified model to study. The system we will work over is the following:

1. We will work with a bidimensional lattice made by small superconducting spheres placed on its sites, all of them with the same radius a and lattice spacing con-

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stant R (distance between the centers of consecutive spheres). This lattice is on the influence of a uniform magnetic field perpendicular to the plane of the lattice.
2. We will neglect the penetration of the magnetic field into the superconducting spheres. This is justified when the separation among sphere' surfaces is greater than a 5 per cent of their radii.
3. Our sample will be always at constant temperature, that means we assume that the lattice is in contact with a good thermalizer so we can neglect the latent heat released by the transition of a granule. In principle it is not difficult to incorporate this effect into the simulation, but as long as we are not interested in the avalanche effect [3] we will assume that the temperature of the lattice is always constant.
4. We will assume that a superconducting grain reaches completely the normal state as soon as at any point on its surface the magnetic field value is greater than B_{sh}. This assumption is certainly correct if the external field strength B_{ex} is larger than B_c as it will be our case.

The magnetostatic interactions between perfectly diamagnetic spheres, placed in an external magnetic field have been studied by U. Geigenmüller and P. Mazur [4] who translated this problem into that of the electrostatic interactions between dielectric spheres; in what follows we shall use their results.

2 Mathematical development

In this section we recall the main results on the magnetic interactions between diamagnetic spheres, which are applied in the numerical simulation. We consider a set of perfectly spherical superconducting grains, all of then with a radius a, placed on the sites of a lattice, since in the space between the superconducting spheres there is no density current and magnetization, the magnetic field $\boldsymbol{B}$ is governed by the two Maxwell equations:

$$\nabla \times \boldsymbol{B}(\boldsymbol{r}) = 0$$
$$\nabla \cdot \boldsymbol{B}(\boldsymbol{r}) = 0 \tag{1}$$

so we can write

$$\boldsymbol{B}(\boldsymbol{r}) = -\nabla U(\boldsymbol{r}) \tag{2}$$

outside the spheres. Equations (1) and (2) imply that the potential U satisfies the Laplace equation.

$$\Delta U(\boldsymbol{r}) = 0 \tag{3}$$

with the following boundary conditions:

1. Due to the Meissner-Oschenfeld effect and the continuity of the magnetic field in the interphase, the normal component of $\boldsymbol{B}(\boldsymbol{r})$ on the surface of the spheres is null, so
$$\left[\frac{\partial}{\partial r} U\left(\boldsymbol{R}_i + \boldsymbol{r}\right)\right]_{r=a} = 0 \tag{4}$$
where $\boldsymbol{R}_i$ is the position of the center of the sphere i and a is the radius of the spheres.
2. If we separate far enough of the spheres, the magnetic field must be the applied one $\boldsymbol{B}_{ex}$
$$\lim_{r\to\infty} -\nabla U(\boldsymbol{r}) = \boldsymbol{B}_{ex} \tag{5}$$

Solving the Laplace equation for the potential U, with this two boundary conditions, we can get a general solution. This is a difficult task which can be made easier in the case where the dominant contributions come from the two-body interactions (which really dominates for dilute or semidilute lattices). In this approximation the field at the surface of each grain is evaluated as a sum of two-body interactions, neglecting those of higher order. As shown in [4], this approximation corresponds to a development in series of the density of microspheres per unit area. In this case, the magnetic field $\boldsymbol{B}_1$ on the surface of a sphere located at the origin when a second one is placed at a distance R along the z direction, with an external magnetic field directed along the positive x direction is:

$$\begin{aligned}\frac{\mid \boldsymbol{B}_1(\theta,\varphi)\mid^2}{\mid \boldsymbol{B}_{ex}\mid^2} &= (\cos\theta\sin\varphi)^2\left[\sum_{q=1}^{\infty}(q-1)!P_q''(\cos\theta)d_q\right]^2 \\ &- (\cos\theta\sin\varphi)^2 \\ &\times\left[\sum_{q=1}^{\infty}(q-1)!\left(P_q'(\cos\theta)+\cos\theta P_q''(\cos\theta)\right)d_q\right]^2 \\ &+\left(\sum_{q=1}^{\infty}(q-1)!P_q'(\cos\theta)d_q\right)^2\end{aligned} \tag{6}$$

where (θ,φ) are the polar coordinates on the sphere 1, $P'(\cos\theta)$ and $P''(\cos\theta)$ are the 1st. and 2nd. derivatives of the Legendre polynomials and d_q are coefficients which depends on the ratio a/R. Equation (6) comes from a multipole expansion and the d_q are determined through a system of linear equations:

$$d_q' = (-1)^{q+1}\frac{(q+1)!}{2q+1}d_q$$
$$d_q' = \delta_{1,q} + \sum_{p=1}^{\infty}\frac{(p+q)!}{p!q!}\frac{pq}{(q+1)(p+1)}\left(\frac{a}{R}\right)^{p+q+1} d_p' \tag{7}$$

The above equations can be solved numerically if one cuts them at a suitable multipole order q_{max} since then (7) is reduced to a system of q_{max} equations. In practice, q_{max} is chosen such that B_1 does not change within the desired accuracy when we retain more than q_{max} terms in (7). For a given accuracy, the value of q_{max} strongly depends on the ratio R/a and increases dramatically for values of $R/a \leq 2.1$. We have checked, in accordance with [4] that for $R/a \geq 2.5$ it is enough to take $q_{max} = 15$ to obtain B_1 with a precision of 0.1 % (see Fig.1).

The contribution of the second sphere to the magnetic field on the surface of the first ($\boldsymbol{B}_{2\to1}$) can be obtained by subtracting to $\boldsymbol{B}_1$ the magnetic field $\boldsymbol{B}_0$ on the surface of an isolated sphere placed on an external magnetic field $\boldsymbol{B}_{ex}$.

$$\boldsymbol{B}_{2\to1} = \boldsymbol{B}_1 - \boldsymbol{B}_0 \tag{8}$$

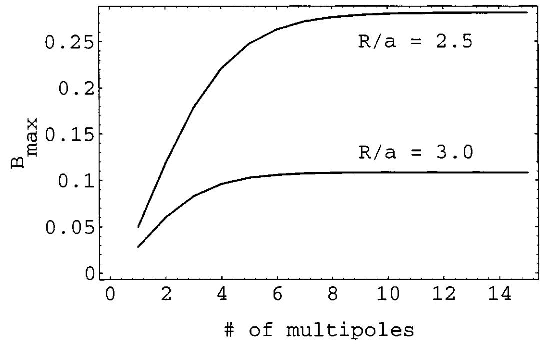

Fig. 1. Maximum value of $B_{2\to 1}$ in units of B_{ex} as a function of the number of multipoles taken into account

Because the problem of the two spheres is invariant under rotations along the external field direction, we can evaluate easily the contribution to the magnetic field on the surface of a given sphere due to the presence of any other, when both are in the plane $y - z$ simply rotating (6) (which means to redefine properly the angles (θ, φ) and use the new value of a/R in (7)).

3 Simulation and results

Limited by the memory capacity and CPU time of our computer, we work in a 100×100 lattice which is enough large to avoid the boundary effects to be important. We have divided the neighbour spheres of a given one in consecutive layers. For a fixed sphere in the lattice, the nearest eight grains constitute the first layer, the next sixteen ones the second layer, the next twenty four ones the third layer and so on (see Fig.2).

Considering the diamagnetic interactions as a sum of two-body interactions, to each value of R/a and for a given error, there is a limiting distance D such that the magnetic field on the surface of a given sphere is not affected, to the desired precision, by the presence of other spheres at a distance greater than D. That means we neglect the interactions between spheres which are faraway enough. In this way, in our simulation, the magnetic field on the surface of each grain is evaluated by adding all the contributions coming from the spheres placed in the nearest three or four layers and then we sum all these contributions with the magnetic field of an isolated sphere; this is made for each grain in the lattice and we perform the following steps:

1. We fix a certain external magnetic field $\boldsymbol{B}_{ex}$ and we evaluate the maximum field strength on the surface of each sphere, taking into account all two-body diamagnetic interactions.
2. We count the number of superconducting spheres which have a field strength larger or equal than B_{sh} in at least a point of their surface.
3. We choose a grain of those before specified in a random way. This grain undergoes the phase transition, and consequently, the magnetic field in the spheres on its neighbourhood decreases. This process simulates the fact that in any realistic situation, not all the spheres will be identical and some imperfections will be randomly distributed.
4. The previous step involves a recalculation of the magnetic field on the surface of all of the neighbouring spheres which interacts with the transited grain.
5. We return to the second step until the number of spheres able to undergo the phase transition, for a given value of the external field is null. In this case, we increase the value of the external field.
6. We make again the steps from two to five until all spheres were in the normal state.

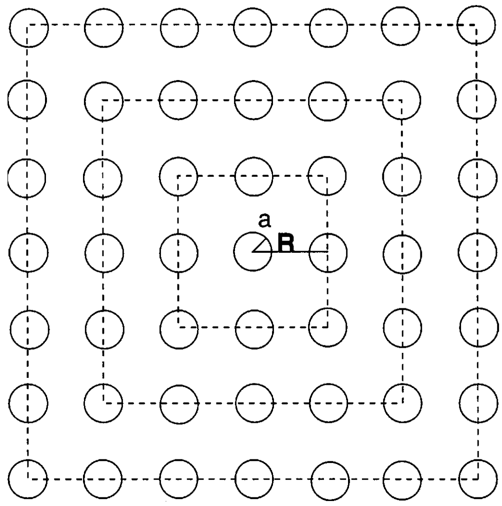

Fig. 2. First three layers surrounding a given sphere

As a result, we get the fraction of superconducting spheres versus the external magnetic field strength. In first place we have done the simulation for a value of $R/a = 2.5$ and considering that each grain only interacts with the spheres placed in the interior of its 2, 3 or 4 neighbour layers (that is, only the interactions of each grain with its 24, 48 or 80 neighbour spheres are taken into account); the results are plotted in Fig. 3 where one can see that the difference between considering three or four layers are of the same order than the statistical errors, so in what follows we will take only three layers; that is, each grain will interact only with its 48 nearest spheres. The results of our simulation are sumarized in Fig. 4 where the fraction F of superconducting spheres is plotted as a function of the external magnetic field B_{ex} for different values of the ratio R/a. The main two points to take into account are:

(a) Instead of a monotonously decreasing curve for the shorter lattice constant R (in units of a) we find three sections, two decreasing ones and between them a plain zone where F is insensitive to small variations of B_{ex}. However for higher values of R/a we get a continuous fall.

(b) The plateau evolves with R/a until its disappearance, but it is always located at $F(B_{ex}) = 0.3$ as it is shown in the plot.

The plateau can be explained, at least qualitatively, in the following way: For higher values of R, we are working

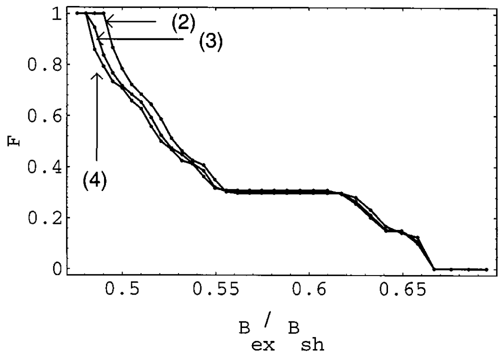

Fig. 3. Fraction of superconducting spheres as a function of the external magnetic field strength, when the interactions of each grain with the spheres placed in the two, three or four nearest layers are considered

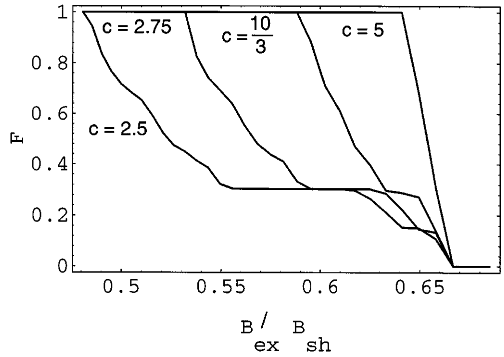

Fig. 4. Fraction of superconducting spheres versus applied field strength for different $c = R/a$ ratios and three layers

in a dilute regime (one sphere is faraway from the others) so the diamagnetic interactions between them are not very important and each time a sphere becomes normal conducting it does not affect very much to the spheres which are located on its neighbourhood; consequently, the plot shows a rapidly decreasing function with increasing field strength (for $R \to \infty$ the plot should approach to a step θ function). At lower values on R, we are working in a more concentrate regime (each sphere is close to its neighbours) so the diamagnetic interactions among them become crucial and when a sphere becomes normal conducting it strongly decreases the maximum field strength on the surface of its neighbours. In this way the falling of the fraction of superconducting spheres with increasing field strength is lower than in the dilute regime, and it forces an effective dilution of the system (a given superconducting grain becomes far and far from its neighbour superconducting spheres).

Between both regimes and interpolating between them, there is the "plateau"; its existence is a consequence of the evolution of the system during the concentrate regime and the fact that the diamagnetic interactions between the spheres are now less important as a consequence of the effective dilution of the system. However, we have not found any theoretical explanation to the fact that the plain zone is always located at $F(B_{ex}) = 0.3$ independently of the initial value of R ($2 < R/a \leq 3.5$); this fact could be related with some property of self-organized systems, a matter which is presently under study.

4 Conclusions

We have done the simulation of a superconducting lattice of micrograins in metastable state in two dimensions put into a perpendicular external magnetic field. In this system we have studied the diamagnetic interactions among the microspheres and we have obtained the ratio of superconducting spheres versus applied magnetic field. These graphics show, for the lower ratios R/a studied (R is the lattice constant and a the spheres' radius), the appearance of a "plateau" while for higher ratios R/a it disappears. We have interpreted the "plateau" as an interpolation state between two qualitatively differents dilution regimes: initially homogenization and, at the end, regular dilution.

From the experimental point of view, the existence of the plateau at small values of R/a is important since it could lower the efficiency of the detector, nowadays this fact has to be proved experimentally and actually it is an open question.

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